ABSTRACT
Biomedical signals are the concentrated expression of human life information and also the window to peep into the phenomenon of life. The study of the theory and methods of biomedical signal detection and processing had great significance for understanding the laws of motion of human life, exploring new disease prevention and treatment methods, as well as the development of high-tech medical equipment industries. Biomedical signal detection and processing almost became the common research area for biomedical engineering. The raw data acquired by Electrical Impedance Tomography (EIT) is the measured boundary voltage data. The boundary voltage is measured as follows: a safe current is applied on the surface of the human body, the voltage on the surface is measured, and the internal impedance distribution and variation is reconstructed via the measured boundary voltage data. For the EIT image reconstruction problem, the measured data contains noise with different levels for some objective reasons, such as the hardware and the measurement way, etc. In this paper, a general framework of subspace method is proposed, and several classic techniques should be induced under this framework. Our experiments are based on real data, and the experimental results show that the subspace method should improve the resolution of the EIT image.


Keywords: electrical impedance tomography, sparse reconstruction, subspace method, denoising

Introduction
Bio-impedance, which carries abundant physiological and pathological information about the human body, facilitates the non-invasive functional evaluation of tissues and organs by bio-impedance technology (5). When a certain disease occurs, functional changes of tissues and organs often precede the organic pathological changes and other clinical symptoms. If these changes could be detected and confirmed in the incubation or functional compensatory periods, it would be highly favorable to the diagnosis, prevention and early treatment of the related disease. Bioelectrical impedance tomography has become a hot topic in the biomedical field at present. Compared with imaging equipment commonly used in hospitals (MRI, CT, X-ray, etc), Electrical Impedance Tomography (EIT) has many advantages. It is portable, non-invasive and non-radiant. It can be used in functional imaging and real-time monitoring. It has a low cost and is easy to operate, which makes it readily accepted by patients and doctors. Therefore, as a potential non-invasive functional imaging technique, EIT has become one of the focuses in the biomedical field. EIT is characterized by functional imaging, in which images are reconstructed according to the impedance change that precedes the morphological or organic changes. Hence, there is research especially devoted to the electrical impedance to the human body (6, 14, 15). In clinical applications (7), this can be used for early detection of breast cancer, real-time dynamic imaging, enables EIT to perform bedside monitoring of thoracic and abdominal bleeding as well as of cerebral hemorrhage. Due to the advantages of being non-invasive and non-radiant, EIT has great clinical value, especially to patients such as pregnant women. Consequently, the research on EIT has fundamental scientific and practical significance.

As shown in Fig. 1, the EIT system consists of two modules (16): a measurement system and an image reconstruction algorithm. After obtaining the measurement data (boundary voltage), image reconstruction and display are performed.

Fig. 1. Electrical impedance tomography system.

For the EIT image reconstruction problem, the measured data contains noise with different levels for some objective reasons, such as the hardware and the measurement way, etc. In order to increase the resolution of the EIT image, the raw data need to be cleaned (3). Nguyen et al. (12) proposed a low-rank approximation method to denoise the MR spectroscopic image, and the image resolution was improved. And Hahn et al. (7) and Kim and Yoo (9) used the Independent Component Analysis (ICA) and Principal Component Analysis (PCA) method to separate the noise and actual signals. In this paper a subspace method was proposed to denoise the raw data recorded by the EIT system.

EIT can be described by partial differential equations (PDEs). As a numerical solution, we get a system of linear equations, namely, a linear matrix expression after discretizing a continuous domain (Fig. 2a) (18):
**Materials and Methods**

**Data acquisition and problem analysis**

In Fig. 2b, there are 16 electrodes which are contained in an EIT system, and the electrodes are coded from 1–16. The data acquisition method is given as follows. Suppose that an opposite sinusoidal drive pattern and adjacent measurement method were applied, i.e., a current was applied on an electrode pair (1, 9), and the voltage measured on (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (10, 11), (11, 12), (12, 13), (13, 14), (14, 15), and (15, 16) electrodes; we get 12 voltages. Similarly, a current should be applied on the pair (2, 10), etc. Finally, 192 = 16×12 voltages were obtained.

To address the critical problem of EIT, which is that the measurement noise has a tremendous impact on the reconstructed images, the measured boundary voltages have to undergo a noise reduction process. There are some existing methods, including PCA, Singular Vector Decomposition (SVD), and Non-negative Matrix Factorization (NMF), etc. Since the measured boundary voltage contains noise, we change Eq. 1 to:

\[
\begin{align*}
Ax &= \tilde{b} \\
\tilde{b} &= b + \epsilon
\end{align*}
\]  

(Eq. 2)

where \( \tilde{b} \) is the measured boundary voltage vector, \( b \) is the real measured boundary voltage (i.e. the data without noise), \( \epsilon \) is the measured noise, \( x \) and \( A \) are the same as above. The measured boundary voltage \( \tilde{b} \) should be considered as the real boundary voltage \( b \) transformed via the following transformation \( B \), i.e.

\[
\tilde{b} = Bb
\]  

(Eq. 3)

Linear expression is needed, such as PCA, SVD, etc.

**General framework of the subspace methods**

The non-matrix general framework (11) of the subspace method is as follows:

\[
x = a_{1}v_{1} + a_{2}v_{2} + \cdots + a_{N}v_{N}
\]  

(Eq. 4)

The matrix term formula is:

\[
x = Av
\]  

(Eq. 5)

Representation methods are needed because they can be used for noise reduction. The typical subspace derivation methods include PCA, ICA, Linear Discriminant Analysis (LDA), etc. The subspace approach (10, 13) uses approximate vectors by projecting them in a low-dimensional subspace. Suppose an original space representation is:

\[
x = a_{1}v_{1} + a_{2}v_{2} + \cdots + a_{N}v_{N},
\]  

where \( v_{1}, v_{2}, \ldots, v_{N} \) are the base in the original N-dimensional space, and there is also a lower-dimensional subspace representation which is on another basis:

\[
x' = c_{1}u_{1} + c_{2}u_{2} + \cdots + c_{K}u_{K},
\]  

where \( u_{1}, u_{2}, \ldots, u_{K} \) is a base in the K-dimensional subspace \((K < N)\). The aim is to find a “natural” set of coordinates for the data. Then, this set could be used to produce a reduced-dimensional representation by linear projection into a subspace. Several criteria have been used for choosing the set of basis vectors (19). To transform a vector \( x \) to the new coordinate system, \( x \) is multiplied by a matrix \( B \) whose rows are the basis vectors \( c = Bx \). The elements of \( c \) could be considered as the values of \( x \) on the new dimensions, or as “coefficients”. In matrix terms, \( x = B^Tc \), since the basis vectors will be columns in \( B^T \), and the expression \( B^Tc \) will give a linear combination of the columns of \( B^T \), using \( c \) as the weights.

As we can represent \( x \) as \( x = B^Tb \), where \( B \) is an orthogonal matrix, we can reformulate this in non-matrix terms as:

\[
x = \sum_{i=1}^{n} (x^T b_i) b_i
\]  

(Eq. 6)

The focus is laid on cases where the basis vectors are chosen so as to depend on the data and are, therefore, informative about the structure of the phenomenon behind the measurements. The idea of the subspace method is to find an approximation of Eq. 6.

**Subspace method derivation**

There is a unique way to represent an arbitrary \( x \) as a linear combination of the new basis vectors \( B \). However Eq. 6 should be truncated after \( k \) terms \((K < N)\):

\[
x = \sum_{i=1}^{k} (x^T b_i) b_i, k < n
\]  

(Eq. 7)

In this way, \( x \) is approximated as a linear combination of some subset of orthogonal basis vectors, and projected into a \( k \)-dimensional subspace. Obviously, each choice of \( k \) orthonormal vectors gives a different approximation. The basic idea of the subspace method is to find the low rank approximation expression of Eq. 6. For more details on the subspace method derivation see below.

**Principal Components Analysis (PCA).** A linear combination of the elements of observation vectors \( x \) that will have maximum variance is found, in other words, a fixed weighting vector \( u \) is found such that (1, 17):

\[
max E\left((xu - E(xu))^2\right)
\]  

(Eq. 8)

This could algebraically be transformed to:
where $R$ is the covariance matrix of $x$, and $u'u = 1$ so that $u$ is of unit length.

Non-negative Matrix Factorization (NMF)

Given an $n$-by-$m$ data matrix $X$ with $X_{ij} \geq 0$ and a pre-specified positive integer $r < \min(n,m)$, NMF finds two non-negative matrices $W$ and $V$ such that (11):

$$X \approx W V^T$$

If each column of $X$ represents an object, NMF approximates it by a linear combination of $r$ “basis” columns in $W$.

Singular Value Decomposition (SVD)

An $m$-by-$n$ real or complex matrix $X$ is a factorization of the form:

$$X = U \Sigma V^T$$

where $U$ is an $m$-by-$m$ real or complex unitary matrix, $\Sigma$ is an $m$-by-$n$ rectangular diagonal matrix with non-negative real numbers on the diagonal, and $V^T$ (the conjugate transpose of $V$) is an $n$-by-$n$ real or complex unitary matrix. The diagonal entries $\Sigma_{jj}$ of $\Sigma$ are the singular values of $X$. The $m$ columns of $U$ and the $n$ columns of $V$ are the left-singular vectors and right-singular vectors of $X$, respectively. The columns of $U$ and $V$ are orthonormal bases. As $U$ and $V^T$ are unitary matrices, the columns of each of these matrices constitute a set of orthonormal vectors which can be considered as basis vectors.

Multidimensional Scaling (MDS)

The distances matrix (kernel matrix) is defined as (8):

$$K = \begin{pmatrix}
\delta_{1,1} & \delta_{1,2} & \ldots & \delta_{1,n} \\
\delta_{2,1} & \delta_{2,2} & \ldots & \delta_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
\delta_{n,1} & \delta_{n,2} & \ldots & \delta_{n,n}
\end{pmatrix}$$

Given $K$, the aim of MDS is to find $n$ vectors $x_1, \ldots, x_n \in \mathbb{R}^N$, such that $\|x_i - x_j\| \approx \delta_{i,j}, i, j \in \{1,2,\ldots,n\}$. That is, MDS (4) seeks to find an embedding from the $n$ objects into $\mathbb{R}^N$ such that distances are preserved. MDS is formulated as an optimization problem, where $x_1, \ldots, x_n$ is found as a minimizer of a cost function:

$$\min_{x_1, \ldots, x_n} \sum_{i < j} \left( \|x_i - x_j\|^2 - \delta_{i,j} \right)^2$$

Then, a solution may be found by general optimization methods.

EIT image reconstruction

We start with a set of boundary measurements expressed as a vector, typically with a large number of dimensions (i.e. each frame has 192 features), find the subspace representation for the raw data $b'$, then we get the clean data $b$ from $b' = Bb$. To find the numerical solution for the linear system $Ax = b$, here we use the L1-Least Square (L1-LS) regularization method (2, 4). The optimization model is:

$$\min_{x_1, \ldots, x_n} \|Ax - b\|^2 + \alpha \|x\|$$

A general optimization solver should be used; here we use the Primal-Dual Interior Point Algorithm. Here, for dynamical imaging, $x$ is the differential imaging result, which means $x = x_2 - x_1$, where $x_1$ is the background conductivity, and $x_2$ is the current conductivity; $x$ is the change of the conductivity.

Results and Discussion

Experimental settings

The physical model experimental settings are given with more detailed information related to the experiments in Table 1.

The Laboratory EIT system which we used in our experiments is shown in Fig. 3. In Fig. 4, the experimental water tank and the reconstruction object are shown. Here we use a copper-bar for testing. A one-dimensional curve of the measured total boundary voltage is shown in Fig. 5. There are two vertical bars in the figure: the first vertical bar represents the frame 32 (background conductivity), and the second bar represents frame 119 (current conductivity). The imaging object is the change in the conductivity.
Experimental results with different subspace methods under different noise levels

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<th>Noise Level</th>
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<th>PCA</th>
<th>SVD</th>
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**TABLE 2**
Experimental results and analysis
The experimental results with different subspace methods (NMF, PCA, SVD, and MDS) under different noise levels (raw data, 100 dB, 90 dB, 80 dB, 75 dB) are shown in Table 2.

The MATLAB code for adding noise is: \( y = \text{awgn}(x, \text{SNR}) \), where \( x \) is the raw data, SNR represents the Signal-to-Noise Ratio of the Gaussian white noise. The unit of the SNR is decibel (dB). The raw data was obtained by the EIT system. We placed the copper-bar into the water-tank, put it in and dragged it out, and then we got voltage sequences via the EIT measurement system. A one-dimensional curve of the measured total boundary voltage (mean value of the measured voltage sequences) is shown in Fig. 5.

From the experimental results, we found that the subspace method should improve the resolution of the EIT image.

Conclusions
One of the reasons for low resolution of the EIT images is the low SNR of the measured data. Therefore, a denoising technique was needed for the raw EIT data. The performed analysis and the obtained experimental results showed that the subspace technique should improve the quality of the EIT image. In our future studies, attention should be paid to the following problems: i. rank parameter \( k \) selection; ii. Construction of a general framework for subspace representation under which more representation methods that nicely fit the EIT data should be employed; iii. Searching for a incremental subspace representation for EIT raw data, which is still a challenging task.

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